PREDICTING LOCOMOTIVE PERFORMANCE

. W. B. Hall. F R Eng., F.I.Mech.E.

The more efficient locomotives tested towards the end of the 'steam era' had overall thermal efficiencies of around 7 or 8 percent when working at optimum conditions; that is, the work done at the drawbar was 7 or 8 percent of the calorific value of the coal burned. When account is taken of the energy losses from the boiler (mainly unburned fuel and heat carried away by the products of combustion) and of mechanical losses between the cylinders and the drawbar, the residual thermal efficiency *referred to the cylinders* could be as high as 14% or 15 %. By comparison, the efficiency of a perfect heat engine operating with similar steam conditions and exhausting to the atmosphere would be around 20%.

The evolution of designs capable of the above performance had proceeded against a background of considerable mechanical ingenuity and engineering insight, but with an almost total lack of a theoretical framework for some of the more important processes involved. Mechanics and to some extent properties of materials were exceptions, but until the early part of the 20th century the theoretical understanding of fluid mechanics, heat transfer and irreversible thermodynamics was not adequate to provide a theoretical framework for crucial aspects of design. Design 'rules' there were in abundance, but many were limited to only small variations from the test data from which they had been derived; most were purely empirical, and some were dimensionally unsound. The situation in the 1930s is well illustrated in a series of articles in the Railway Gazette by E.L.Diamond [1] which reviews both the nature of the problem and the many attempts to produce theoretical guidelines for design purposes. Even with the development of subjects crucial to the understanding of cylinder performance, such as fluid mechanics and heat transfer, the sheer complexity of the processes defeated close analysis; the solution by hand of highly non-linear equations by numerical methods was painfully slow, and not a weapon that commended itself to a firmly practical branch of the engineering profession.

The first signs of a way out of the dilemma appeared in a paper by W.A.Tuplin [2] in 1950. Experiment had proceeded apace, and reliable and detailed measurements were becoming available from stationary test plants, and from the well controlled road tests pioneered by S.O.Ell [3] which formed the basis for subsequent British Railways reports on locomotive performance. Tuplin set himself the task of producing a theoretical indicator diagram as the basis for predicting cylinder power and adopted a method of analysis which characterised performance in terms of dimensionless groupings of the many variables involved - a technique pioneered by Osborne Reynolds in the 1890s. He did not of course have the benefit of a high speed computer to integrate the steam flow equations and had to rely on an ingenious but somewhat intuitive reconstruction of the indicator diagram; in his words the theoretical treatment "was distinguished more for convenience than for rigour". He validated his methods against test data for several modern superheated locomotives with moderate success. The choice of superheated locomotives was important since there is strong evidence to suggest that superheat can drastically reduce heat transfer effects in the cylinders and so eliminate the disastrous consequences of cylinder condensation; this considerably simplifies the problem. His methods do not appear to have been adopted in the industry - perhaps those who might have benefited were too busy coping with reconstruction and standardisation following the war, or perhaps they just mistrusted the academic approach! In any event, there proved to be too little time before the end of the 'steam era' to make further progress.

The advent of the digital computer has transformed the process of dealing with complex non-linear problems; this, together with an improved understanding of compressible fluid flow has made it possible to develop Tuplin's approach and to rapidly investigate changes in the main design parameters. It is now possible to set up a mass balance for the steam in the cylinder and to integrate this around the cycle; pressure drops suffered by the steam in flowing through the ports are accounted for, as are the limitations introduced by sonic flow. The net amount of steam admitted, and the mean effective pressure are computed, thus enabling a prediction to be made of performance characteristics such as Indicated Horsepower, Indicated Tractive Effort, Steam Consumption and Efficiency. It must be re-emphasised that the solution only applies to situations where heat transfer between the steam and the cylinder and valves can be discounted: that is, to engines operating with sufficient superheat. The definition of 'sufficient' in this context remains to be determined following further work on the theory. The method of solution is set out below and predictions are compared with the data provided by tests on a British Railways Class 7 4-6-2 locomotive (Britannia).

An outline of the analysis

The aim is to calculate the pressure as a function of the volume of steam in the cylinder - that is, to reproduce a theoretical "indicator diagram". This is easy if it can be assumed that whilst the steam port is open the steam pressure in the cylinder remains constant at the pressure in the steam chest, and whilst the exhaust port is open the pressure in the cylinder is atmospheric. However, even at quite slow speeds the resistance to flow through the ports is significant and must be accounted for. Mostly, the effect of this resistance is disadvantageous, but it can occasionally improve performance. A case in point is the effect of exhaust port resistance when running at short cut-off and therefore early release: with a high resistance, thus retaining at least some steam in the cylinder until the end of the stroke, one can extract more work and improve efficiency. A typical comparison between indicator diagrams with and without pressure losses in ports is shown in Fig.1.

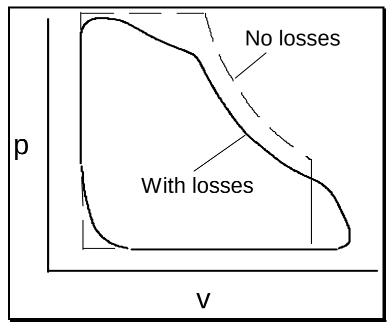


Fig.1 Indicator diagrams

Since the valves constitute the major source of flow resistance one must first calculate the port openings as a function of crank angle for any specified cut-off. The choice is between an accurate numerical solution based on a particular valvegear and an approximate solution in which, for example, angularity effects in parts of the valve gear are neglected; these effects are small for a well designed Walschaerts gear. Connecting rod angularity effects are larger and produce an asymmetry between the fore and back strokes; however these to some extent cancel out over one revolution. In the following analysis I have accounted for connecting rod angularity but not valve gear angularities. Port openings can thus be expressed as functions of crank angle and con-rod/stroke ratio, and are determined by cut-off and by specified characteristics of the valvegear like the ratio of lead to lap, and port width to lap.

The next step is to establish a method of calculating the rate of flow through the ports for specified pressure differences across them. (i.e. between steam chest and cylinder in the case of the inlet ports, and between cylinder and exhaust passages in the case of the exhaust ports). Compressible flow equations must be used, and choking of the flow when it reaches the speed of sound must be correctly modelled. I have used "polytropic" expansion relationships with expansion indices appropriate to reversible adiabatic flow of superheated steam in the case of the inlet and exhaust ports. (The condition of the steam at exhaust depends upon working conditions; fluid friction can result in the steam still being superheated at exhaust. However, the calculation procedure allows the expansion index to be varied in different parts of the cycle). Whilst reversible flow equations are used to establish the flow rate, the overall process including the friction and turbulence downstream of the ports is correctly modelled as one of constant enthalpy. A "coefficient of discharge" can be applied to the flow so as to model the contractions of the flow caused by sharp edges etc. These relationships, together with that expressing the port opening, enable the flow through the ports to be established in terms of the pressure difference across them.

The final stage is to write down a differential equation expressing the pressure balance in the cylinder. This must balance the flow into or out of the cylinder with the expansion of the steam already there; when integrated around the cycle it yields the indicator diagram. Since the initial conditions at the start of the process (i.e. with the piston at the end of the cylinder) are not known ***a priori*** the process must be repeated over at least two cycles. In fact , further repeats are required to establish the pressure in the blast pipe, which depends of course on the average flow from the exhaust ports of all the cylinders. In writing the equations I have tried wherever possible to cast them in a non-dimensional form; this has the advantage that the results can be expressed in terms of a number of dimensionless groups instead of a much larger number of individual variables. Details of the analysis are given in Appendix 1.

Numerical solution of the equations

Details of the computer programme devised to solve the equations are given elsewhere[4], but it may be helpful at this stage to give a brief description of the method. As mentioned above the equations are, as far as possible, cast and solved in dimensionless form; however the computer programme accepts dimensional data and produces results in units that are familiar in steam locomotive engineering.

The first step is to accept input data from the keyboard or from a stored file, provision being made for amending values as required. (These data are in fact converted into SI units for the purpose of calculation, but the user need not be aware of this, since the results are converted back to conventional units before presentation). Valve data (e.g. cut-off, lap, lead etc.) are then used to compute the flow area through the inlet and exhaust ports as a function of crank angle. Other input data are used to evaluate dimensionless groups such as G in Equations (12) and (13) in Appendix 1.

As described above, the equations representing the pressure balance in the cylinder are then integrated to yield the pressure around one revolution of the crank. During this process it is necessary to check when the port is fully open and to use this as the flow area if the valve overrides the port. It is also necessary to check when the pressure ratio across the port exceeds the critical pressure ratio for sonic (choked) conditions and thus to use the appropriate formulation for calculating the flow rate. Finally, as explained above, the integration must be repeated until the pressure at the blastpipe is established. The result is a theoretical indicator diagram which takes account of wiredrawing but omits any effects caused by heat transfer between the steam and the surfaces of valve and cylinder.

Comparison with Test Data

Stationary Plant (Rugby) tests and Controlled Road tests on a British Railways Class 7 4-6-2 Mixed Traffic locomotive (‘Britannia’) are reported in Ref.[5]. Sufficient information is provided for a comparison of test data with the predictions of overall performance (e.g. Indicated Horsepower, Indicated Tractive Effort and Indicated Efficiency) and for a limited comparison of Indicator Diagrams. The tests used a “Farnboro” indicator which presents the cylinder pressure as a function of crank angle rather than piston displacement; unfortunately only a limited sample of diagrams is included in the report. I have used an effectively infinite con-rod/stroke ratio for the overall performance predictions since in this case an approximate average of the angularity effects for the for and back strokes is required; the correct ratio has been used to compare indicator diagrams, and the prediction applies to the front end of the cylinder.

Input data (Britannia)

The following data were used in the calculations (unless variations are specifically stated):

Steam lap	1.688 in.	Boiler pressure	250 psig
Exhaust lap	0 in.	Steam temperature	350 °C
Lead	0.25 in.	No. of cylinders	2
Port width	2.25 in.	Blastpipe diameter	5.375 in.
Port perimeter	25.16 in.	Expansion index	1.3
Cylinder diameter	20 in.	Compression index	1.3
Stroke	28 in.	Discharge coefficient	0.75
Wheel diameter	74 in.	Clearance/swept vol.	0.103
Ratio conrod/stroke	10000 (i.e. angularity effects eliminated, IHP model)		
	4.57 (for Indicator Diagram comparisons)		

Indicated H P

The numerical integration produces the 'dimensionless mean effective pressure' = mep/p_o (where p_o is the steam chest pressure) which can be inserted in the well worn formula for horsepower as follows:

I H P = PLAN/33000 = (mep/p_o) *x* p_o *x stroke x piston area x (strokes/min) /33000*

Fig.2 shows the IHP test results (indicated by the curves taken directly from Ref.5) as a function of speed at four different values of cut-off. The predicted values calculated by the methods of this paper are shown as points. Two disposable quantities in the calculation are the steam chest pressure and the coefficient of discharge for flow through valves and ports. Specific data on steam chest pressure are not given in the report, although it is stated that "the majority of the work was performed with full regulator opening". I have taken the average steam chest pressure to be 230 psig and the discharge coefficient to be 0.75. There is, of course, an element of 'curve fitting' in this choice of discharge coefficient in that I find it gives the best agreement between prediction and test. However, the value is certainly quite plausible and gives confidence that the method will provide reasonably accurate trends of performance when parameters are varied from the standard configuration. A further test of this choice will be made when predicted and measured indicator diagrams are compared (see below)

The magnitude of the wire-drawing effect can be judged by calculating the IHP that would be achieved in its absence. In the case of 25% cut-off at 60 mph this would be 2700 hp instead of 1600 hp. At higher speeds output is also limited by the occurrence of sonic velocity in the blastpipe nozzle (the so-called 'front end limit') if not also by the capacity of the boiler to produce more steam.

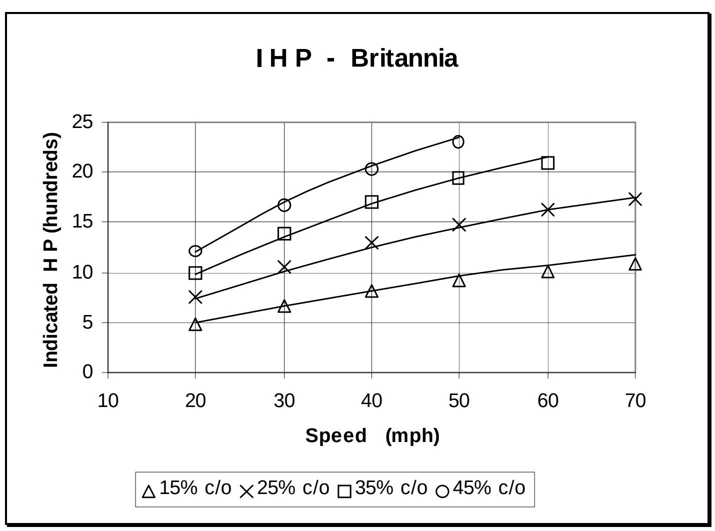


Fig.2 Comparison of Britannia IHP with predictions. Lines represent 'smoothed' test data (Ref.5) and points are predictions using the theory in this paper.

Steam Consumption

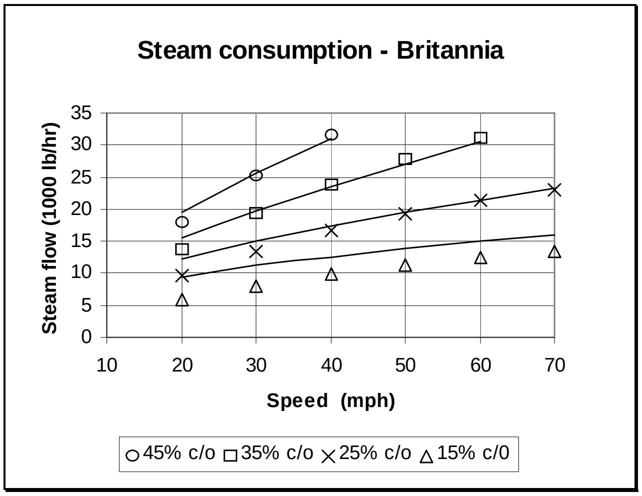


Fig.3 Comparison of Britannia Steam consumption with predictions. Lines represent 'smoothed' test data and points are predictions.

Measured and predicted steam consumptions are compared in Fig.3. The agreement is quite good at the higher speeds and longer cut-offs, but the measured steam flow is significantly greater at low speed and for the complete speed range with 15% cut-off. The disposable constants 'steam chest pressure' and 'discharge coefficient' which were used to produce a good fit with the IHP data could not be adjusted to reduce this discrepancy, and one must conclude that there are genuine physical reasons why the theory is inadequate.

The most probable reasons are heat transfer between steam and cylinder, or leakage of steam past the valve or piston: the problem is to distinguish between these two mechanisms. Higher speed mean less time for heat transfer, but also less time for leakage. It is likely that early cut-off would involve more heat transfer because of the greater ratio of heat transfer surface (cylinder, cover and piston) to steam volume during admission, but leakage on the other hand might be reduced because of the lower mean effective pressure. It seems therefore that the trend of the results supports the heat transfer hypothesis rather than leakage.

The surfaces of piston and cylinder are alternately exposed to exhaust steam and the superheated inlet steam and the cylinder block might be expected to be at an intermediate mean temperature. The surface temperature will fluctuate above and below this mean, and whilst the fluctuation penetrates only a few millimetres below the surface, a significant amount of heat may be stored or released. The suggestion is that the surfaces are at their lowest temperature immediately prior to admission, and that some steam is condensed and stored in the surface layer during admission, thus resulting in a greater flow through the valves during this period. The water layer so produced would then re-evaporate as the steam pressure falls during expansion, thus cooling the surface layer. Such a mechanism was recognised during the controversy surrounding the 'Missing Quantity' (see for example Perry [6] writing in 1909); it can account for large discrepancies between measured and predicted steam consumption in the case of unsuperheated engines, and whilst the effect is greatly reduced with superheat it may not be entirely eliminated.

Indicator diagrams

Ref.[5] includes four sample indicator diagrams taken with a Farnboro type indicator. This instrument records on a drum rotating at a speed proportional to the engine speed, so the diagram is one of pressure as ordinate against crank angular rotation as abscissa. These have been converted to the conventional form of pressure against piston displacement. Unfortunately the diagrams were changed in size when Ref.[5] was printed, so it was not possible to scale the pressure ordinate; however all tests were made at full regulator so I have

assumed that they were all at the steam chest pressure of 230 psig. (as for Figs.2 and 3). The following diagrams (Fig.4)
compare the theoretical predictions (bold lines) with points taken from the diagrams in Ref.[3] (circles).*)* ***(Please note that in contrast with Figs. 2 and 3 the bold lines here represent the theoretical predictions)*** The value of valve discharge coefficient used in the theoretical prediction is 0.75, as was also used in obtaining the data for Figs. 2 and 3. Several other values were tried, but none gave a significant improvement over this value. It was found that the predicted exhaust backpressure calculated on the basis of a 5.375″ diameter blast nozzle was consistently lower than that shown on the test diagrams if a blast nozzle discharge coefficient of 1.0 was assumed. A discharge coefficient of 0.85 produced closer agreement but even after this correction, the predicted backpressure still appears low in the case of 15% and 45% cut-off. It may be that this is a consequence of the simplified model of valve motion used; whilst this correctly models connecting rod angularity it does not account for valvegear angularity effects although these are usually small with a well designed Walschaerts gear.

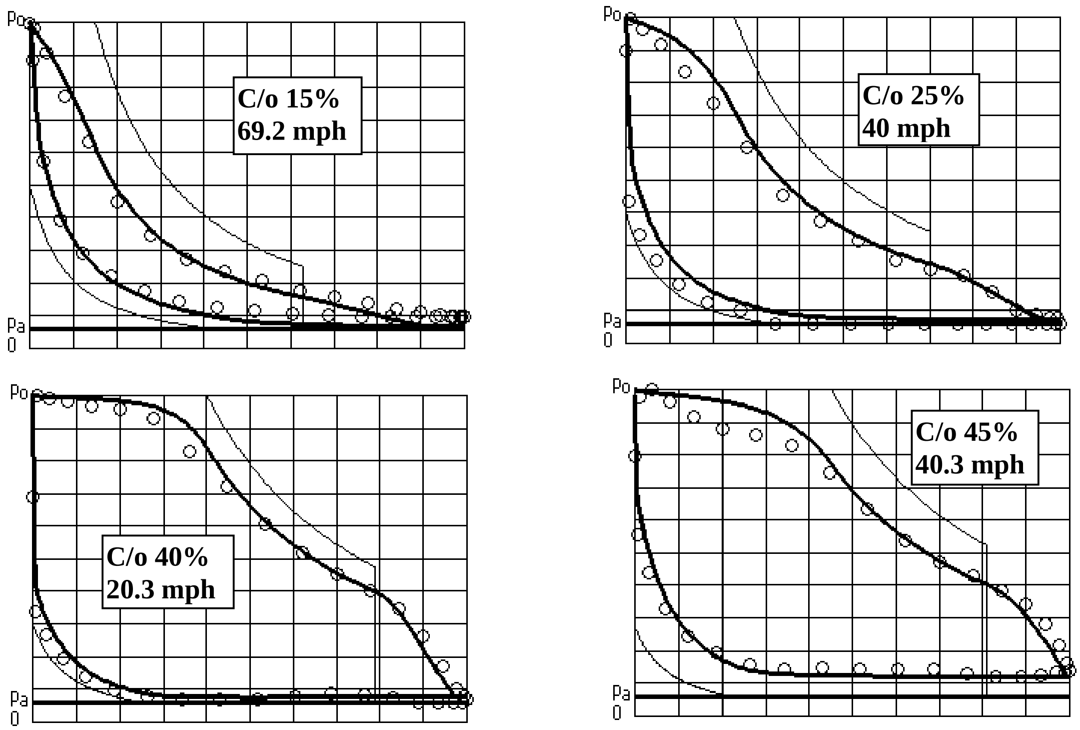


Fig.4 Theoretical and Experimental Indicator Diagrams

Agreement between predictions and test results is generally good. The 25% cut-off result appears to be somewhat out of line with the others, and leads me to wonder whether the location of top dead centre has been correctly defined on the indicator record. In all cases there appears to be a significantly greater decline of pressure during admission than theory predicts - particularly for the 45% cut-off diagram (a very heavy steam consumption). I suspect that this effect may be due to a reduction in the steam chest pressure during admission - something that the theory does not simulate. I hope to remedy this defect, and also to seek data on measured steam chest pressures since I understand that the steam chest was indicated in the tests. However, the overall agreement seems good enough to justify use of the theoretical predictions in examining such effects as changes in steam and exhaust lap, lead, clearance volume, blast pipe diameter, boiler pressure etc. These can be examined quickly using the associated computer programme.

Some examples of predictions

Effect of Lead

Lead, the amount by which the inlet valve is open at dead centre, is introduced so that there is no undue delay in achieving full steam pressure at the beginning of the stroke. It is most important at high speed and short cut-off, when it can have a major effect on the shape of the indicator diagram and on power output. Fig.5 illustrates this by comparing indicator diagrams for normal lead and for zero lead at a cut-off of 15% and 80 mph (Data for Britannia)

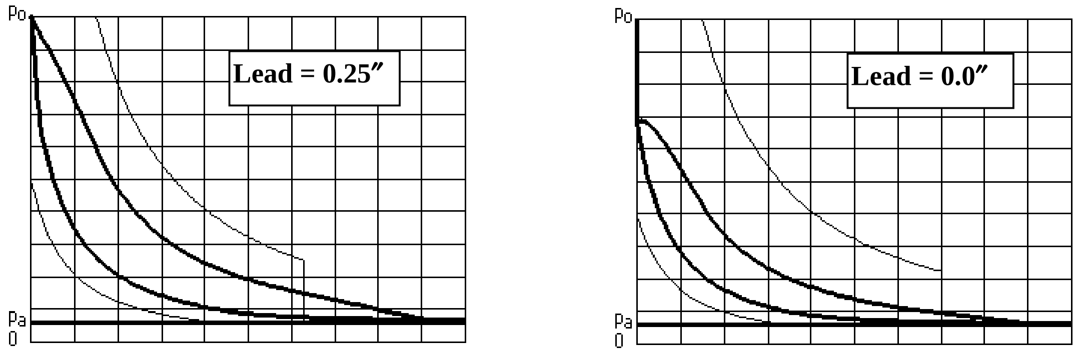


Fig.5 Predicted effect of Lead on Britannia at 15% cut-off and 80 mph

The greatly reduced area of the diagram with no lead results in a decrease in IHP from 1120 to 716, and in Indicated Tractive Effort from 5250 lb. to 3358 lb. Indicated Efficiency decreases from 16.1% to 15.5%, a smaller change since the steam flow is reduced as well as the mean effective pressure. For lower speeds and longer cut-offs the effect of changes of lead is minimal, as is illustrated in Fig.6, which refers to Britannia at 50% cut-off and 20 mph.

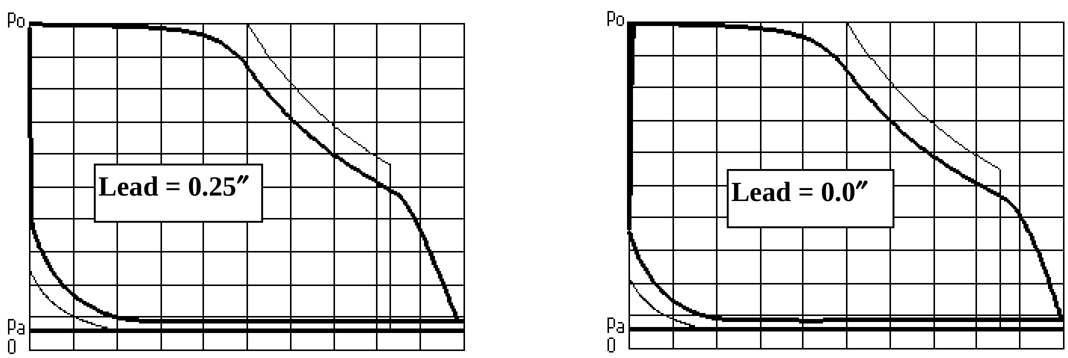


Fig.6 Predicted effect of Lead on Britannia at 50% cut-off and 20 mph.

Too much lead can cause difficulties in starting, so the ideal arrangement would seem to be that in which lead is increased as cut-off is shortened and speed increases. Stephenson valve gear has this characteristic; Walschaerts gear on the other hand is a constant lead gear.

The Effect of Steam Lap

An increase in steam lap necessitates a greater valve travel and results in larger port openings for a given cut-off setting. Consequently wiredrawing is reduced. The effect is more noticeable when the speed is high and the cut-off short. Fig.7 illustrates the effect on Britannia at 20% cut-off and 70 mph.

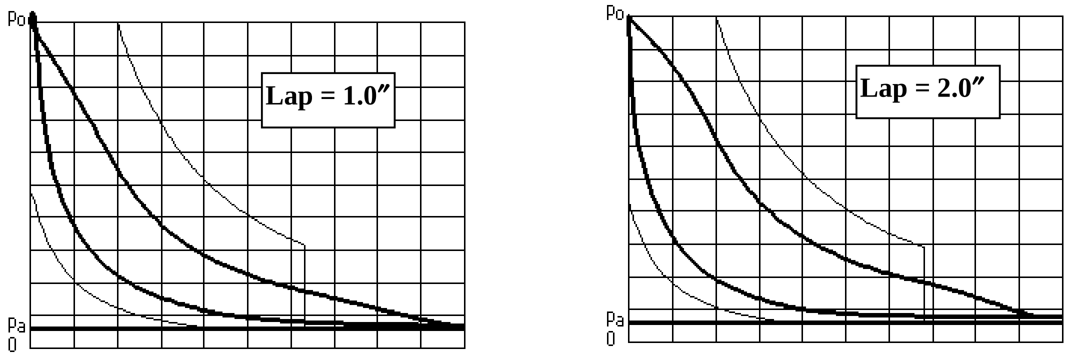


Fig.7 Predicted effect of Steam Lap on Britannia at 20% cut-off and 70 mph

In this example an increase in steam lap from 1″ to 2″ increases the IHP from 1126 to 1494, and the Indicated Tractive Effort from 6036 lb. to 8000 lb. There is a commensurate increase in steam flow, so the Indicated Efficiency does not change significantly.

The Effect of Blastpipe Nozzle Diameter

The pressure drop through the blastpipe nozzle raises the exhaust pressure at the cylinders and thus reduces the mean effective pressure and power output. Opening up the blastpipe reduces the pressure drop, but also reduces the velocity of the jet and the vacuum produced in the smokebox. The control loop linking the steam flow through the blastpipe with the draught, with the air flow induced through the fire, and thus with the rate of steam production is sometimes referred to as the Stephensonian Cycle.

It is not obvious that a single nozzle diameter is best for all operating conditions, and indeed variable nozzles have been tried. A rather serious limitation occurs when the flow through the nozzle reaches sonic velocity: some Great Western locomotives were fitted with a 'jumper' which opened automatically in response to blastpipe pressure and thus increased the effective nozzle size. In the present analysis I have assumed a fixed nozzle diameter and a coefficient of discharge C_b . It will be seen in the Appendix that from its definition a change in C_b is proportional to the change in nozzle cross sectional area. As mentioned in connection with the comparison of measured and predicted indicator diagrams, a value of 0.85 seems appropriate in this case.

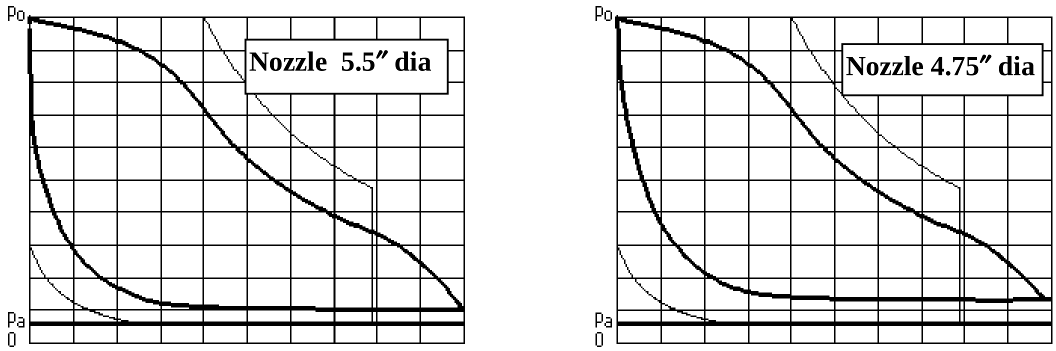


Fig.8 The effect of blast nozzle size on cylinder backpressure

Fig.8 shows indicator diagrams corresponding to blast nozzles of 4.75″ and 5.5″ in the case of Britannia at 40% cut-off and 50 mph. The backpressure with a 5.5″dia. nozzle is 12.2psig, and that with a 4.75″ dia. Nozzle is 20.3psig; the corresponding IHPs are 2220 and 2010. In the case of the smaller nozzle the flow would be sonic, and the Indicated Efficiency of the engine would be reduced from 13.2% (with the larger nozzle) to 12.6%

Conclusions

A theoretical model of steam flow through locomotive cylinders and blastpipe has been constructed and solved using numerical methods. Compressible flow relationships are used, and the limitations imposed by sonic flow through valves and blastpipe are accounted for. The predictions of the model have been compared with published data for the tests carried out on the British Railways Standard Class 7, 4-6-2 Mixed Traffic Locomotive (Britannia), and the two sets of data are found to be in substantial agreement in terms of both overall performance, and indicator diagrams.

Comparison with the test data has revealed a possible deficiency in the theory in that it does not predict such a rapid decline in pressure in the cylinder as the tests show during admission, particularly when the load on the engine is great. A possible reason for this is the assumption in the theory that the steam chest pressure remains constant, whereas it probably declines somewhat during the admission phase. The theoretical treatment should really model the cylinder and the steam chest as coupled systems; means of introducing this change are being investigated.

The theory does not yet account for heat transfer between the steam and the valves and cylinder walls. The good agreement with Britannia data confirms that, as expected, the use of superheated steam largely inhibits such heat transfer; nevertheless it would be useful to develop the theory so that it could deal with unsuperheated steam. Condensation in unsuperheated engines can result in drastically reduced efficiency, and was the object of considerable debate at the end of the 19th century. The effects are worst at low speeds and with small engines and can result in two or three times the expected steam consumption. It appears that the heat of condensation is absorbed by the cylinder during admission and released back into the steam during the later stages of expansion and during exhaust. Many of the theoretical ideas associated with this process were formulated by John Perry [6] but were too difficult to apply without the use of numerical methods and a computer. That constraint is now gone, but there are still problems in accurately assessing condensation rates.

The numerical methods used to solve the model are embodied in a computer programme [4] written in the 'C' language; this can be made available to anyone who wishes to investigate locomotive performance (within the above mentioned limitations). The programme solves the differential equation that defines the pressure balance in the cylinder and produces an 'indicator' diagram and a summary of various performance characteristics such as Indicated Horsepower, Tractive Effort, and Indicated Efficiency.

Acknowledgements

My attention was drawn to the work by Professor Tuplin by Cedric Iliffe: this started it all! Access to the data on the 'Britannia' tests, and much additional help was provided by Phil Atkins of the National Railway Museum, York. Much helpful advice and criticism came from my friends Tom Jones and Duncan Webster.

References

1. Diamond, E.L. "The Horse-power of Locomotives – its calculation and measurement", The Railway Gazette, April 12, 1935.
2. Tuplin, W.A. "Locomotive Cylinder Power" . The Engineer, Feb 10, 1950.
3. Ell, S.O. "Developments in Locomotive Testing". J.Inst. Loco. Engrs. Paper No 527 ,1953.
4. Computer programme based on the theory advanced in this paper. Source code and *.exe* file is available from the Author.
5. British Railways Bulletin No.5 "Performance and Efficiency Tests with Exhaust Steam Injector" (BR Class 7 4-6-2 Mixed Traffic Locomotive)
6. Perry, J. "The Steam Engine and Gas and Oil Engines". Macmillan, 1909

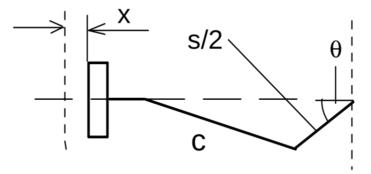
APPENDIX 1 : DETAILS OF THE ANALYSIS

Motion of Piston and Valve

Piston

The piston displacement, x and the dimensionless displacement ξ are given by:

$$\xi = x/s = c/s + 0.5 - \sqrt{(c/s)^2 - (\sin\theta/2)^2} - \cos\theta/2 \quad . . (1)$$



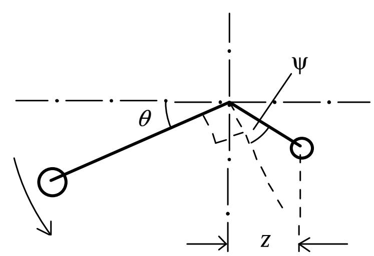
where c = length of connecting rod
s = stroke

(When determining overall performance characteristics that depend upon both strokes of the piston angularity effects are eliminated by assuming c/s to be very large.)

Valve

Assume the "equivalent eccentric" model, in which the valve motion is completely determined by the steam lap L, the lead l, and the crank angle at cut-off θ_c . The port openings are determined by these quantities together with the exhaust lap and the port width. The equivalent eccentric is, as its name implies, an eccentric with a radius r_{eq} and angle of advance ψ such that it represents the motion of the valve; as the real valve gear is "notched up" this radius and angle of advance will change. We must therefore relate r_{eq} and ψ to the independent variables L, l, and θ_c. (Usually it is the fractional cut-off ξ_{co} that is specified rather than the value of the crank angle at cut-off. However the two are related by Equation (1)

(Equation (1) is inverted using a root-finder in the computer programme to give θ as a function of ξ_{co})



The valve displacement from its mid position is

$$z = r_{eq}\,\mathbf{sin}(\theta + \psi) \quad . . . . (2)$$

At front dead centre $(\theta = 0)$, the inlet port is open by an amount l, so that :

$$L + l = r_{eq}.\mathbf{sin}\,\psi$$

Also, at cut-off, when the crank angle is θ_c , the displacement of the valve from its mid-point is L, so that: $L = r_{eq}.\mathbf{sin}(\theta_c + \psi)$

It follows from these two conditions that :

$$\psi = \mathbf{tan}^{-1}\left\{ \mathbf{sin}\,\theta_c \big/ \left[L/(L+1) - \mathbf{cos}\,\theta_c \right] \right\} \quad \text{and} \quad r_{eq} = (L + l)/\mathbf{sin}\,\psi \quad . . . . (3)$$

With these definitions, the port openings can be expressed as:

Inlet port opening , $y_i = r_{eq}.\mathbf{sin}(\theta + \psi) - L$ . . . .(4)

Exhaust port opening , $y_o = -r_{eq}.\mathbf{sin}(\theta + \psi) - L_{ex}$ . . . . (5)

The maximum opening must of course be limited to the actual width of the port.

Flow through the ports

Admission phase

During admission the pressure upstream of the valve is assumed to remain constant at the steam chest pressure. At the start of a stroke the cylinder pressure is usually fairly close to the steam chest pressure, but as the piston accelerates the cylinder pressure falls and the speed of flow of steam through the valve increases. If the valve is regarded as an orifice discharging into a vessel (the cylinder) in which the pressure is uniform, one can calculate the instantaneous speed of flow of the steam into the cylinder. As the cylinder pressure drops, so the steam speed through the valve increases; in some cases it may reach the speed of sound so that the flow becomes choked and any further decrease in cylinder pressure does not affect the speed. (What this really means is that there is no way that messages from downstream can be transmitted upstream of the orifice to tell the steam to speed up; such messages travel at the speed of sound and can therefore make no headway against the sonic velocity of the steam at the throat of the orifice). This phenomenon occurs at a particular value of the ratio of downstream to upstream pressure (the 'critical pressure ratio'): for superheated steam the critical pressure ratio is about 0.55. The calculation must therefore take account of the fact that no further increase in steam speed is possible if the pressure ratio falls below this value. Having found the steam speed we can then use the upstream density and the port opening from Equation (4) to calculate the mass flow rate of steam into the cylinder.

Calculation of the mass flow is straightforward enough but we need to know the volume of steam admitted - corresponding to its specific volume in the cylinder. The specific volume after admission, when the steam is virtually at rest can be determined by assuming that the overall process, of discharge through the valve followed by stagnation, is one of constant enthalpy. For steam in the range of conditions of interest, constant enthalpy is quite well represented by a constant value of the product pv, so that if the upstream pressure and specific volume are p_o and v_o and the cylinder pressure is p, the specific volume of the steam in the cylinder will be $p_o v_o / p$. The flow rate through the valve is determined by the ratio of the upstream pressure to the pressure at the 'throat' of the valve. For subsonic flow this pressure is the same as the cylinder pressure, p; for sonic flow it will be a pressure (greater than p) determined by the upstream pressure and the critical pressure ratio. Thus we define the throat pressure, p', so that when the flow is choked it is equal to the critical throat pressure for sonic flow; otherwise it is equal to the cylinder pressure. The following equations are based on standard relationships for reversible adiabatic flow of a fluid that follows the expansion law $pv^n = constant$. (Note that whilst overall flow process is irreversible, the flow to the 'throat' will approach reversibility). The variable cross sectional area of the inlet port is denoted by A_i, and the coefficient of discharge by C_d.

$$\textit{Mass flow rate}, M = C_d \,.\, A_i \,.\, \frac{1}{v_o}\left(\frac{p'}{p_o}\right)^{\frac{1}{n}} \,.\, \sqrt{2\left(\frac{n}{n-1}\right) p_o v_o \left\{1-\left(\frac{p'}{p_o}\right)^{\frac{n-1}{n}}\right\}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\textit{Specific volume in cylinder} = v_o\, p_o \,/\, p$$

$$\textit{Volume flow rate to cylinder} = C_d \,.\, A_i \,.\, \sqrt{p_o v_o}\left(\frac{p_o}{p}\right)\sqrt{2\frac{n}{n-1}\left\{\left(\frac{p'}{p_o}\right)^{\frac{2}{n}} - \left(\frac{p'}{p_o}\right)^{\frac{n+1}{n}}\right\}} \quad \ldots\ldots\ldots\ldots\ldots \mathbf{(7)}$$

where:

p = cylinder pressure, p_o = steam chest pressure, p' = throat pressure,

$p' = p$ when $p > p_c$ and
$p' = p_c$ when $p < p_c$

$$\frac{p_c}{p_o} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$ (the 'critical pressure ratio' for flow through the inlet valve)

Exhaust phase

This is as for the Admission phase, except that in this case the pressure upstream of the valve is the pressure in the cylinder, which of course is not constant. On the other hand the pressure downstream is constant. The specific volume of the steam in the cylinder is obtained by assuming that the expansion in the cylinder follows the law pv^n = *constant*. This is not strictly true because of the irreversibilities in the expansion through the inlet port; however the discrepancy introduced affects only the estimation of velocity and is not thought to be significant. Choking may occur if the exhaust pressure is less than the critical pressure for sonic flow through the exhaust ports. As with flow through the inlet valve we must distinguish between the downstream pressure and the critical pressure in determining the flow velocity. For flexibility, the calculation also allows a different expansion index *m* for flow through the exhaust ports; this index also applies during the compression phase. The cross sectional area of the exhaust ports is A_o

$$Mass\ \ flow\ rate,\ M = C_d \, . \, A_o \, . \, \frac{1}{v}\left(\frac{p'}{p}\right)^{\frac{1}{m}} \sqrt{2\left(\frac{m}{m-1}\right) pv \left\{1 - \left(\frac{p'}{p}\right)^{\frac{m-1}{m}}\right\}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

And since the specific volume of steam leaving cylinder at pressure p is $M \times v$,

$$Volume\ \ flow\ rate\ of\ \ steam\ leaving\ cylinder = C_d \, . \, A_o \sqrt{p_o v_o} \left(\frac{p}{p_o}\right)^{\frac{n-1}{2n}} \sqrt{2\frac{m}{m-1}\left\{\left(\frac{p'}{p}\right)^{\frac{2}{n}} - \left(\frac{p'}{p}\right)^{\frac{m+1}{m}}\right\}} \quad ..(9)$$

where

p = cylinder pressure, p_e = steam chest pressure, p' = throat pressure

$p' = p_e$ when $p_e > p$

$p' = p_c$ when $p_e < p_c$

$$\frac{p_c}{p} = \left(\frac{2}{m+1}\right)^{\frac{m}{m-1}}$$ (the 'critical pressure ratio' for flow through the exhaust valve)

The Pressure Balance equation

The next stage of the analysis is to calculate the change in cylinder pressure following a small movement of the piston, thus leading to a differential equation which can be integrated to yield the pressure - volume diagram for a complete cycle. The increase in volume behind the piston is filled partly by the steam flowing in through the inlet port, and partly by the expansion of the steam that is already in the cylinder. We have already calculated the former quantity. The latter can be obtained by the following argument. Suppose the volume of steam in the cylinder changes by an amount δV and the pressure by an amount δp; the steam already in the cylinder will expand by an amount $m(\partial v/\partial p)_s.\delta p$ where m is the mass of steam in the cylinder, and the partial derivative of v is at constant entropy, s. Thus the volume of new steam required is given by:

$$\delta V - m(\partial v/\partial p)_s.\delta p = \delta V - (V/v)(\partial v/\partial p)_s.\delta p$$

For a reversible polytropic expansion with an expansion index n, $(\partial v/\partial p)_s = (1/n)(v/p)$. Using this and substituting the inflow of steam from Equation (6), we get the pressure balance equation for an interval of time δt (corresponding to δp and δv) in the form:

$$C_c.A_i.\sqrt{p_o v_o}\,\frac{p_o}{p}\sqrt{2\frac{n}{n-1}\left\{\left(\frac{p'}{p_o}\right)^{\frac{2}{n}}-\left(\frac{p'}{p_o}\right)^{\frac{n+1}{n}}\right\}}.dt = dV - \frac{V}{np}dp \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

Rearranging and changing the variables to φ $(= p/p_o)$ and crankangle θ, and writing the angular velocity $\omega = d\theta/dt$, and $\phi' = p'/p_o$

$$\frac{d\phi}{d\theta} = \left[C_d A_i \sqrt{p_o v_o}.\frac{1}{\omega\phi V}\sqrt{\frac{2}{n-1}\left\{(\phi')^{\frac{2}{n}}-(\phi')^{\frac{n+1}{n}}\right\}} - \frac{1}{V}\frac{dV}{d\theta}\right]n\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

The quantities $(dV/d\theta)$ and V can be evaluated from Equation 1, from which

$$\frac{V}{A_p s} = \frac{V_{cv}}{A_p s} + \left[c/s + 0.5 - \sqrt{(c/s)^2 + (\mathbf{sin}\,\theta/2)^2} - \mathbf{cos}\,\theta/2\right] = \xi_{cv} + \xi$$

$$\frac{1}{A_p s}\frac{dV}{d\theta} = \left[\frac{\mathbf{sin}\,\theta.\mathbf{cos}\,\theta}{4\sqrt{(c/s)^2 + (\mathbf{sin}\,\theta/2)^2}} + \mathbf{sin}\,\theta/2\right] = \frac{d\xi}{d\theta}$$

where V_{cv} = clearance volume, A_p = piston area, and $\xi_{cv} = V_{cv}/A_p s$

The flow area through the valve A_i can be evaluated from the port opening (Equation (4) for inlet and Equation (5) for exhaust) and the perimeter of the ports, making due allowance for bridging.

A similar analysis can be carried out for the exhaust phase. The final version of the two forms of the differential equation are given below.

Admission

$$\frac{d\phi}{d\theta} = \frac{n\phi}{\xi_{cv}+\xi}\left\{G.\varsigma_i\,\frac{1}{\phi}\sqrt{\frac{2}{n-1}\left[(\phi')^{2/n}-(\phi')^{n+1/n}\right]} - \frac{d\xi}{d\theta}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

where the dimensionless port opening, $\varsigma_i = \frac{y_i}{L} = \frac{r_{eq}}{L}.\mathbf{sin}(\theta+\psi) - 1$

and $\phi' = \phi$ when $\phi \geq \left(2/n+1\right)^{n/n-1}$ (i.e. subsonic flow through port opening)

$\phi' = \left(2/n+1\right)^{n/n-1}$ when $\phi \leq \left(2/n+1\right)^{n/n-1}$ (i.e sonic flow through port opening)

Exhaust

$$\frac{d\phi}{d\theta} = \frac{-m\phi}{\xi_{cv} + \xi} \left\{ G.\varsigma_o \phi^{\frac{n-1}{2n}} \sqrt{\frac{2m}{n(m-1)} \left[(\phi'')^{2/m} - (\phi'')^{\frac{m+1}{m}} \right]} + \frac{d\xi}{d\theta} \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

where the dimensionless port opening, $\varsigma_o = \frac{y_o}{L} = \frac{r_{eq}}{L}.\mathbf{sin}(\theta + \phi) - \frac{L_{ex}}{L}$

and $\phi'' = \phi_e / \phi$ when $\phi_e / \phi \geq \left(2/m+1 \right)^{m/m-1}$ (i.e. subsonic flow through port opening)

$\phi'' = \left(2/m+1 \right)^{m/m-1}$ when $\phi_e / \phi \leq \left(2/m+1 \right)^{m/m-1}$ (i.e sonic flow through port opening)

In Equations (12) and (13) $G = \frac{C_d}{\omega} \frac{(valve\_perimeter).L}{A_p s} \sqrt{np_o v_o}$

and $\phi_e = p_e / p_o$

Equation (12), as well as serving the admission phase, also serves the expansion phase because the valve closes at cut-off, thus making y_i zero. The equation will then be found to reduce to the statement that $dp/dV = -np/V$, which integrates to the expansion law pv^n*.=constant*. Similarly, Equation (13) serves the compression phase as well as the exhaust phase.

Equations (12) and (13) represent a first order ordinary differential equation defining the dimensionless pressure ϕ as a function of crank angle θ. In the accompanying computer programme (written in C and compiled for a PC) this equation is integrated using a Runge-Kutta routine in which the step length is automatically varied to maintain a specified accuracy. The initial pressure at the start of the stroke is unknown because even when the inlet port is open the cylinder pressure may not rapidly come into equilibrium with the steam chest pressure. The procedure adopted is to start the first integration around the cycle by assuming that the cylinder pressure is equal to the steam chest pressure; completion of this cycle then provides a suitable initial condition for the second integration.

The process as described above has assumed implicitly that the exhaust pressure is known. In reality there will be a pressure drop through the “front end”, and the magnitude of this drop will change with the rate of flow of steam through the blastpipe. This effect can be simulated by repeating the integration beyond the two cycles described above, calculating the total steam flow and the blastpipe pressure drop after each integration and using this to determine the exhaust pressure for the next integration. This is then repeated until there is no significant change in the diagram. The method of calculating the blastpipe pressure drop is described in the Appendix.

The RK routine is capable of integrating several first order differential equations simultaneously, providing the opportunity to integrate the pressure and also the flow through the inlet valve. The former leads directly to the Mean Effective Pressure for the cycle, and the latter to the Steam Consumption.

APPENDIX 2 : BLASTPIPE PRESSURE

The main programme calculates the steam flow through the cylinders, so the mass flow rate through the blastpipe, M, is known. In order to determine the pressure ratio across the blast nozzle (of cross sectional area A_b) we can use a relationship such as Equation 6 which was used on the ports. Denoting the blastpipe pressure as p_e, and the atmospheric pressure as p_a we can write the relationship in the form:

$$\frac{M}{A_b} = p_a \sqrt{\frac{2n}{n-1}} \frac{1}{\sqrt{p_e v_e}} \sqrt{\left(\frac{p_e}{p_a}\right)^{\frac{2(n-1)}{n}} - \left(\frac{p_e}{p_a}\right)^{\frac{n-1}{n}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (15)$$

We wish to solve this in order to establish a relationship between M/A_b and the ratio p_e / p_a but unfortunately the quantity $p_e v_e$ is not known. The way out is to recognise that over the pressure range 0.1 to 0.2 MPa it can, to a good approximation, be represented as a function of the exhaust enthalpy h_e ; this in turn can be evaluated from the initial enthalpy and the work done in the cylinders (First Law analysis). Thus:

$$f(h_e) = p_e v_e = -550.6 + 0.298 h_e - 0.0000106 h_e^{\,2} \text{ kJ/kg} \quad \ldots\ldots\ldots\ldots \ (16)$$

With $\sqrt{p_e v_e}$ known, Equation 15 can be recognised as a binomial in $\left(p_e / p_a\right)^{n/n-1}$ and solved as:

$$\frac{p_e}{p_a} = \left[\frac{1}{2} + \sqrt{\frac{1}{4} + \beta^2}\right]^{n/n-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (17)$$

where $\beta = \frac{M}{A_b} \frac{\sqrt{f(h_e)}}{p_a} \sqrt{\frac{n-1}{2n}}$

If the pressure ratio exceeds a critical value the flow through the blastpipe will reach sonic speed; beyond this ratio the mass flow will depend only upon the upstream pressure p_e .The critical pressure ratio in terms of the upstream pressure p_e and the throat pressure in the blast nozzle p_t is given by:

$$\frac{p_t}{p_e} = \left(\frac{2}{n+1}\right)^{n/n-1}$$

If this is inserted in Equation (6) in place of p'/p_o we obtain the following relationship:

$$p_e = \frac{M}{C_b A_b} \Bigg/ \sqrt{\frac{n}{f(h_e)} \cdot \left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (18)$$

Depending upon whether the pressure ratio across the nozzle exceeds the critical ratio for sonic flow, Equation 17 or Equation 18 is used in the numerical solution to determine the backpressure suffered by the cylinders. Equation (17) or Equation (18) can now be used in conjunction with the pressure balance equation for the exhaust phase, Equation (13), since the pressure p_e at inlet to the blastpipe may be taken to be the same as the exhaust pressure seen by the cylinders. Iteration is required since the exhaust pressure is initially taken to be atmospheric. The first pass through the pressure balance equation yields the mass flow m and the exhaust enthalpy h_e which enable the exhaust pressure to be revised using Equation (17) or Equation (18). This revised value is then used in the second pass through the pressure balance equation, and this process is repeated until the exhaust pressure stabilises.